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STABILIZATION OF A GIVEN POSITION OF AN ELASTIC ROD*

E.K. LAVROVSKII and A.M. FORMAL'SKII

An elastic rod which can be displaced along a straight line in a plane is considered. The rod is put into motion by an electric motor mounted at one end. A weight is clamped at the other end. The stabilization of the position of the rod by linear feed-back is studied. The control voltage fed to the motor is a linear combination of signals for the displacement of the rod, for the rate of displacement, for its integral, and for the deformations. Delay in the control circuit is taken into account. Domains of asymptotic stability are constructed in the space of feed-back factors. This problem arises, for example, when an elastic manipulator is designed.

1. The equations of motion. Consider an elastic homogeneous rod of length l and constant cross-section S, which can be displaced in the horizontal plane (the plane of Fig.1). The motion of the rod is controlled by an electric drive. The motor ME and the drive reducing gear R are mounted in the carriage C. The output pinion of R engages with the straight toothed director-rake DR. As a result, the carriage can move along the director. The fixed axis OZ is parallel to the director. The end of the rod O is cantilever clamped in the carriage, so that the straight line OX, tangential to the neutral line of the rod at the point O, is perpendicular to the director. At the other end of the rod a weight is clamped, which we shall regard as a particle of mass M.

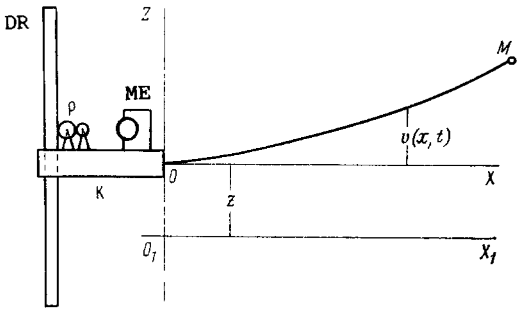

Fig.1

In Fig.1, instead of the rod, we show its (curved) neutral line OM. We assume that OM always lies in the horizontal plane. Denote by $v(x, t)$ the deviation of the point with coordinates x of the neutral line at the instant t from the moving axis OX. When there is no deformation, $v(x, t) \equiv 0$, and the lines OM, OX coincide.

Let z denote the deviation (displacement) of the end of the rod O from the wanted fixed point O_1. In the wanted position the rod neutral axis also coincides with the fixed axis O_1X_1, perpendicular to O_1Z.

In the framework of the linear theory of thin straight non-extensible rods /1, 2/ the equations of notation of this mechanical system can be written as

$$EIv''''(x, t) + \rho S[v^{\cdot\cdot}(x, t) + z^{\cdot\cdot}(t)] = 0 \tag{1.1}$$

$$M_K z^{\cdot\cdot}(t) = F - EIv'''(0, t) \tag{1.2}$$

$$v(0, t) = v'(0, t) = v''(l, t) = 0$$

$$EIv'''(l, t) - M[v^{\cdot\cdot}(l, t) + z^{\cdot\cdot}(t)] = 0 \tag{1.3}$$

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Here, ρ is the rod material density, E is Young's modulus, I is the constant moment of inertia of the rod cross-section about the vertical axis, M_k is a sum whose terms are the mass of the carriage plus electric drive and the moments of inertia, reduced in the light of the reduction factor, of the motor armature and the reduction gear pinion (which naturally have the dimension of mass), and F is the force acting from the rake on the reduction output pinion, created by the moment of the electromagnetic forces about the armature axis.

Eq.(1.1) describes the plane transverse oscillations of the rod /1-3/, given the acceleration $z^{\cdot\cdot}(t)$ of the end O. The equation takes no account of energy dissipation during the oscillations. The drive carriage motion is described by Eq.(1.2), the second term on the right-hand side of which is the shearing force acting on the carriage from the rod.

If the right-hand end is clamped to the centre of mass of the weight, and the moment of inertia of the weight about the centre of mass $J \neq 0$, the boundary condition $v''(l, t) = 0$ has to be replaced by $Jv^{\cdot\cdot\prime}(l, t) = -EIv''(l, t)$.

The rod rotary motion about an end was studied in /4-6/ (see also L.D. Akulenko, S.A. Mikhailov, and O.L. Satovskaya, Dynamic models of elastic manipulation robots, Preprint No. 349, Inst. Problem Mekhaniki, Akad. Nauk SSSR, Moscow, 1988). Though the linearized equations of motion in these papers differ from (1.1)-(1.3), they have the same structure.

Neglecting the motor winding inductance, we write the force F as /7/:

$$F = d_1 u - d_2 z^{\cdot} \tag{1.4}$$

Here, u is the control voltage fed to the motor, and d_1, d_2 are positive constants whose values depend on the motor characteristics and the reduction factor.

We introduce the new variable

$$w(x, t) = v(x, t) + z(t) \tag{1.5}$$

which characterizes the total deviation of the deformed rod from the straight line O_1X_1. We also introduce the dimensionless variables w^*, x^*, z^*, t^*, u^* in accordance with

$$w = lw^*,\ x = lx^*,\ z = lz^*,\ t = \tau t^*,\ u = Uu^* \tag{1.6}$$

$$\tau^2 = \rho S l^4/(EI),\quad U = EI/(l^2 d_1)$$

Substituting (1.4)-(1.6) into Eqs.(1.1)-(1.3) and omitting the asterisks, we obtain

$$\begin{gathered} w''''(x, t) + w^{\cdot\cdot}(x, t) = 0 \\ (\mu w^{\cdot\cdot}(0, t) + dw^{\cdot}(0, t) + w'''(0, t) = u \\ w'(0, t) = w''(1, t) = 0,\quad w'''(1, t) = mw^{\cdot\cdot}(1, t) \\ \left(\mu = \frac{M_k}{\rho S l},\ m = \frac{M}{\rho S l},\ d = \frac{d_2 l}{\sqrt{EI\rho S}}\right) \end{gathered} \tag{1.7}$$

Here, μ and m are the dimensionless masses of the carriage plus drive and the weight, and d is the dimensionless factor of counter-electromotive force. The second relation of (1.7), obtained from Eqs.(1.2), (1.4) with the aid of the equation

$$w(0, t) = z(t) \tag{1.8}$$

plays the role of boundary condition in the new boundary-value problem. The a posteriori Eq.(1.8) is obtained from the boundary condition (1.3) $v(0, t) = 0$. If the control u is independent of the variable z, the latter can be regarded as cyclical.

2. *Formulation of the problem and the control.* If $u \equiv 0$, the boundary value problem (1.7) has the solution

$$w(x, t) = C\quad (v(x, t) = 0,\ z = C) \tag{2.1}$$

where C is an arbitrary constants. The solution (2.1) corresponds to an undeformed rod, deviating by $z = C$ from the line O_1X_1. If $C = 0$, we obtain from (2.1)

$$w(x, t) = 0\quad (v(x, t) = 0,\ z = 0) \tag{2.2}$$

Let us find the control u that ensures asymptotic stability of the solution (2.2).

The stabilizing control will be sought as the linear feedback

$$Tu^{\cdot}(t) + u(t) = -\gamma_0 w(0, t) - \gamma_1 w^{\cdot}(0, t) - \gamma_2 \int_0^t w(0, \zeta)\, d\zeta - \sum \sigma_n w''(x_n, t) \tag{2.3}$$

Here, $T > 0$ is the dimensionless time constant, $\gamma_0, \gamma_1, \gamma_2$ are the constant feedback factors with respect to z and its derivative and integral, σ_n is the constant feedback factor with respect to the bending deformation of the rod at the point x_n, while throughout, $n = 1, \ldots, N$ and summation is performed from $n = 1$ to $n = N$. We take $t = 0$ as the start of the control process. The feedback (2.3) assumes that position-, rate of change of position-, and deformation-sensers (strain gauges) are present.

Corresponding to the linear bundary-value problem (1.7),(2.3) we have a spectrum of eigenvalues. We consider the following specific form of our problem on the asymptotic stability of the solution (2.2):
it is required to find, in the space of feedback factors (2.3), the domain in which all the eigenvalues λ are such that $\operatorname{Re}\lambda < 0$.

The problem of finding the control of systems with distributed parameters for which the eigenvalues λ are such that $\operatorname{Re}\lambda \leqslant 0$ was studied in /8, 9/. For the different definitions of asymptotic stability of the motion of systems with distributed parameters see e.g., /10, 11/.

Apart from (1.7), we naturally take for comparison the equations of motion of an absolutely rigid rod with a weight at one end the feedback (2.3). In the dimensionless variables of (1.6), these equations are

$$(\mu + m + 1)z^{\cdot\cdot} + dz^{\cdot} = u$$
$$Tu^{\cdot} + u = -\gamma_0 z - \gamma_1 z^{\cdot} - \gamma_2 \int_0^t z(\zeta)\,d\zeta \tag{2.4}$$

3. *The characteristic equation.* The solution of problem (1.7), (2.3) will be sought in the form

$$w(x, t) = Le^{\lambda t}X(x)$$

where L is a constant, λ is an eigenvalue, and $X(x)$ an eigenfunction.

For $X(x)$ we obtain the boundary value problem

$$X''''(x) + \lambda^2 X(x) = 0 \tag{3.1}$$

$$[\mu\lambda^2 X(0) + d\lambda X(0) + X'''(0)](T\lambda + 1)\lambda =$$
$$(\gamma_0\lambda + \gamma_1\lambda^2 + \gamma_2)X(0) - \lambda\sum\sigma_n X''(x_n) \tag{3.2}$$
$$X'(0) = X''(1) = 0, \quad X'''(1) = m\lambda^2 X(1)$$

When $\gamma_2 = 0$, both sides of the first of Eqs.(3.2) must be cancelled by λ

The solution of problem (3.1), (3.2) will be sought as the sum

$$X(x) = C_1 e^{\nu x} + C_2 e^{i\nu x} + C_3 e^{-\nu x} + C_4 e^{-i\nu x} \tag{3.3}$$

where $C_1, \ldots, C_4$ are unknown constants, i is the square root of -1, and

$$\lambda^2 = -\nu^4 \tag{3.4}$$

On substituting (3.3) into condition (3.2), we obtain a system of linear homogeneous equations in the constants $C_1, \ldots, C_4$. The non-zero eigenvalues λ satisfy the equation

$$\det\begin{Vmatrix} 1 & i & -1 & -i \\ e^{\nu} & -e^{i\nu} & e^{-\nu} & -e^{-i\nu} \\ (1+m\nu)e^{\nu} & (-i+m\nu)e^{i\nu} & (-1+m\nu)e^{-\nu} & (i+m\nu)e^{-i\nu} \\ a_+ & b_- & a_- & b_+ \end{Vmatrix} = 0 \tag{3.5}$$
$$a_\pm = R(\lambda) \pm \nu^3(T\lambda + 1)\lambda + \nu^2\lambda\sum\sigma_n e^{\pm\nu x_n}$$
$$b_\pm = R(\lambda) \pm i\nu^3(T\lambda + 1)\lambda - \nu^2\lambda\sum\sigma_n e^{\mp i\nu x_n}$$
$$R(\lambda) = \lambda^2(\mu\lambda + d)(T\lambda + 1) + \gamma_0\lambda + \gamma_1\lambda^2 + \gamma_2$$

Expanding the determinant in (3.5), we obtain

$$R(\lambda)R_1(\nu, m) - \nu^3\lambda(T\lambda + 1)R_2(\nu, m) + {}^1/_2\nu^2\lambda\sum R_3(\nu, x_n, m) = 0 \tag{3.6}$$
$$R_1(\nu, m) = Q_1(\nu) + m\nu Q_-(\nu), \quad R_2(\nu, m) = Q_+(\nu) + 2m\nu Q_2(\nu)$$
$$Q_1(\nu) = 1 + \cos\nu\,\operatorname{ch}\nu, \quad Q_2(\nu) = \cos\nu\,\operatorname{ch}\nu$$
$$Q_\pm(\nu) = \operatorname{sh}\nu\cos\nu \pm \operatorname{ch}\nu\sin\nu$$
$$R_3(\nu, x, m) = \operatorname{sh}\nu\sin[\nu(1-x)] - \operatorname{ch}\nu\cos[\nu(1-x)] +$$
$$\cos\nu\,\operatorname{ch}[\nu(1-x)] + \sin\nu\,\operatorname{sh}[\nu(1-x)] + \operatorname{ch}\nu x - \cos\nu x +$$
$$2m\nu\{\operatorname{sh}[\nu(1-x)]\cos\nu + \operatorname{ch}\nu\sin[\nu(1-x)]\}$$

The polynomial $R(\lambda)$ is characteristic for the electric motor with feedback (2.3) when $\sigma_n = 0$, while $R_1(\nu, m)$ is the characteristic quasipolynomial of the elastic rod with cantilever clamping, and $R_2(\nu, m)$ is the characteristic quasipolynomial of the rod whose left end can move freely along the straight director, and the tangent to the neutral line of which at the point O is perpendicular to the director.

Two expressions for λ are found from Eq.(3.4): $\lambda = i\nu^2$ and $\lambda = -i\nu^2$. However, if ν is the root of one equation, then $i\nu$ is the root of the other, so that the λ found by solving the equations are the same. It therefore suffices to analyse just one equation, obtained by substituting into (3.6) e.g., the expression

$$\lambda = i\nu^2 \tag{3.7}$$

In addition to ν Eq.(3.6), (3.7) has the roots $-\nu, i\bar{\nu}$, and $-i\bar{\nu}$, i.e., the roots are grouped in fours. Corresponding to the roots $\pm\nu$ and $\pm i\bar{\nu}$ we have the eigenvalues λ and $\bar{\lambda}$.

4. Domains of stability. To construct the domains of asymptotic stability in parameter space, we shall use the method of D-divisions /12/. If λ takes imaginary values, ν takes imaginary of real values.

Let

$$\sigma_n = 0 \tag{4.1}$$

We substitute $\nu = \varepsilon$ into Eqs.(3.6) and (3.7),where ε is a real number, and we equate the real and imaginary parts to zero

$$\begin{gathered}(\mu T\varepsilon^8 - d\varepsilon^4 - \gamma_1\varepsilon^4 + \gamma_2)R_1(\varepsilon, m) + T\varepsilon^7 R_2(\varepsilon, m) = 0 \\ (-\mu\varepsilon^6 - dT\varepsilon^6 + \gamma_0\varepsilon^2)R_1(\varepsilon, m) - \varepsilon^5 R_2(\varepsilon, m) = 0\end{gathered} \tag{4.2}$$

These define, in parameter space, mappings of the $\nu = \varepsilon$ real axis, $-\infty < \varepsilon < +\infty$. They remain unchanged when ε is replaced by $-\varepsilon$, or by $\pm i\varepsilon$. This is because the roots of Eqs.(3.6) and (3.7) are grouped in fours, as mentioned above. It follows from what has been said that the boundary of an asymptotic stability domain in parameter space belongs to the surface (4.2), obtained with $0 \leqslant \varepsilon < \infty$. We shall construct the domains analytically by passing from special cases to the general case.

First, let

$$\gamma_2 = 0, \quad T = 0 \tag{4.3}$$

With $\gamma_2 = 0$, both sides of (4.2) have to be cancelled by ε^2. On putting then $\varepsilon = 0$, we obtain $\gamma_0 = 0$. With $\varepsilon > 0$, from the first of Eqs.(4.2) we obtain $\gamma_1 = -d$, and from the second

$$\gamma_0 = \mu\varepsilon^4 + \varepsilon^3 R_2(\varepsilon, m)/R_1(\varepsilon, m) \tag{4.4}$$

It can be shown that, on the semi-axis $\varepsilon > 0$, the zeros of the functions $Q_1(\varepsilon)$ and $Q_-(\varepsilon)$ alternate, as do the zeros of $Q_+(\varepsilon)$ and $Q_2(\varepsilon)$. Given any m, in an interval between zeros of opposite signs of $Q_1(\varepsilon)$ and $Q_-(\varepsilon)$ ($Q_+(\varepsilon)$ and $Q_2(\varepsilon)$), there is just one zero of $R_1(\varepsilon, m)$ ($R_2(\varepsilon, m)$). The zeros of $R_1(\varepsilon, 0)$ and $R_2(\varepsilon, 0)$ equal to $Q_1(\varepsilon)$ and $Q_+(\varepsilon)$, alternate. The zeros of $R_1(\varepsilon, m)$, $R_2(\varepsilon, m)$ vary continuously as m varies. It can be shown that these functions have no common zeros for any m.

By what has been said, given any m, the zeros of $R_1(\varepsilon, m)$ and $R_2(\varepsilon, m)$ alternate. Hence, in particular, as ε varies from 0 to $+\infty$ the quantity (4.4) varies from $-\infty$ to $+\infty$. The boundaries of the domain of stability thus include the lines $\gamma_0 = 0$ and $\gamma_1 = -d$. To find the domain, we write the approximate characteristic Eqs.(3.6), (3.7) in the neighbourhood of $\gamma_0 = 0$, $\gamma_1 = -d$, $\nu = \lambda = 0$. For this, we put $\gamma_0 = -\Delta_0$, $\gamma_1 = -d - \Delta_1$, $\lambda = \Delta\lambda$. Series-expanding the left-hand side of Eqs.(3.6), (3.7) retaining only the leading terms, we obtain

$$-\Delta\lambda^2(\mu + m + 1) + \Delta\lambda\Delta_1 + \Delta_0 = 0 \tag{4.5}$$

With $\Delta_0 = 0$, $\Delta_1 > 0$, and likewise with $\Delta_0 > 0$, $\Delta_1 = 0$, this equation has a real root $\Delta\lambda > 0$.

By what has been said, the domain of asymptotic stability, if there is one, is given by the inequalities

$$\gamma_0 > 0, \quad \gamma_1 > -d \tag{4.6}$$

Asymptotic stability in fact occurs in this domain, call it D.

To prove this, we use a similar device to that in /3, 13/. Using the two penultimate boundary conditions (3.2), we obtain

$$\int_0^1 X''''(x)\bar{X}(x)\,dx = X'''(1)\bar{X}(1) - X'''(0)\bar{X}(0) + \int_0^1 X''(x)\bar{X}''(x)\,dx$$

In this equation we substitute the derivative $X''''(x)$ from Eq.(3.1) and the derivatives $X'''(1)$, $X'''(0)$ from the first and last of conditions (3.2). Then,

$$\lambda^2\left[\int_0^1 X(x)\overline{X}(x)\,dx + \mu X(0)\overline{X}(0) + mX(1)\overline{X}(1)\right] +$$

$$\lambda(\gamma_1 + d)X(0)\overline{X}(0) + \gamma_0 X(0)\overline{X}(0) + \int_0^1 X''(x)\overline{X}''(x)\,dx = 0$$

In the domain D the roots of this quadratic equation in λ are negative. Hence all the eigenvalues λ are such that $\operatorname{Re}\lambda \leqslant 0$. But on the other hand, under conditions (4.6), there are no eigenvalues λ such that $\operatorname{Re}\lambda = 0$. The assertion is proved.

In the problem of stabilizing the position $z = 0$ of an absolutely rigid rod, in accordance with Eqs.(2.4) under condition (4.3), the domain of asymptotic stability is again given by inequalities (4.6). hence, given position and velocity feedback, the "pliability" (or "compliance") of the rod does not lead to loss of stability.

Consider the more general case than (4.3), when only

$$\gamma_2 = 0 \tag{4.7}$$

Putting $\varepsilon = 0$ in Eqs.(4.2), then $\varepsilon > 0$, we see that the boundary of the stability domain consists of pieces of the straight line $\gamma_0 = 0$ and of the curve

$$\gamma_0 = dT\varepsilon^4 + \mu\varepsilon^4 + \varepsilon^3 R_2(\varepsilon, m)/R_1(\varepsilon, m) \tag{4.8}$$
$$\gamma_1 = -d + \mu T\varepsilon^4 + T\varepsilon^3 R_2(\varepsilon, m)/R_1(\varepsilon, m)$$

As $\varepsilon \to 0$, we see from (4.8) that $\gamma_0 \to 0,\ \gamma_1 \to -d$. Let $\varepsilon_s = \varepsilon_s(m)\ (s = 1, 2, \ldots)$ be the s-th root of $R_1(\varepsilon, m)$. For $0 < \varepsilon < \varepsilon_1$ we have $R_1(\varepsilon, m) > 0,\ R_2(\varepsilon, m) > 0$, i.e., $\gamma_0 > 0,\ \gamma_1 > -d$. As $\varepsilon \to \varepsilon_1 - 0$ we have $\gamma_0, \gamma_1 \to \infty$. It follows from Eqs.(4.8) that

$$\gamma_1 = T\gamma_0 - d(1 + T^2\varepsilon^4) \tag{4.9}$$

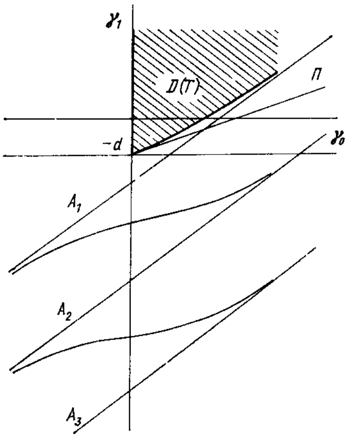

Fig.2

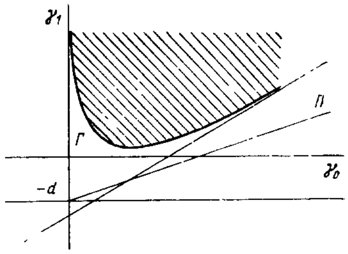

Fig.3

Hence, as $\varepsilon \to \varepsilon_1$, the curve (4.8) tends to the asymptote A_1

$$\gamma_1 = T\gamma_0 - d(1 + T^2\varepsilon_1^4) \tag{4.10}$$

Since the zeros of $R_1(\varepsilon, m)$, $R_2(\varepsilon, m)$ alternate, the curve (4.8) lies in the strip between its asymptotes A_1 and A_2 for $\varepsilon_1 < \varepsilon < \varepsilon_2$. The asymptote A_2 is given by (4.9) with $\varepsilon = \varepsilon_2$. As $\varepsilon \to \varepsilon_1 + 0$, we have $\gamma_0 \to -\infty$, and the curve (4.8) tends to the asymptote A_1 (4.10). As $\varepsilon \to \varepsilon_2 - 0$, we have $\gamma_0 \to \infty$, and the curve (4.8) tends to the asymptote A_2. The branch of the curve (4.8), obtained with $\varepsilon_s < \varepsilon < \varepsilon_{s+1}$ $(s = 2, 3, \ldots)$, is located between the asymptotes A_s and A_{s+1} (Fig.2). The curve (4.8) thus consists of an infinite number of branches, each of which cuts the γ_1 axis.

Consider the first branch of (4.8), obtained with $0 < \varepsilon < \varepsilon_1(m)$. Let $D(T)$ (Fig.2) be the open domain bounded by this branch and the semi-axis $\gamma_0 = 0,\ \gamma_1 \geqslant -d$. With $\gamma_0, \gamma_1 \in D(T)$, there is no eigenvalue such that $\operatorname{Re}\lambda = 0$. As $T \to 0$ we have $D(T) \to D$, and if $\gamma_0, \gamma_1 \in D$, then all the eigenvalues are such that $\operatorname{Re}\lambda < 0$. On further considering the set $D(T)$ with $0 \leqslant T < \infty$ in the space of the three parameters γ_0, γ_1, T, we can see that, if $\gamma_0, \gamma_1 \in D(T)$, asymptotic stability holds. It can be shown by suitable arguments that, if $\gamma_0, \gamma_1 \notin D(T)$, the system is unstable. In short, the domain of asymptotic stability has been obtained in the case (4.7).

The stability domain of the equilibrium $z = 0$ of system (2.4) with $\gamma_2 = 0$ lies between the semi-axis $\gamma_0 = 0, \gamma_1 \geqslant -d$ and the half-line Π:

$$\gamma_1 = -d + \frac{T(\mu + m + 1)}{Td + \mu + m + 1}\gamma_0, \quad \gamma_0 > 0 \tag{4.11}$$

The curve (4.8) touches the line (4.11) at the point $\gamma_0 = 0, \gamma_1 = -d$, and lies above it for $\gamma_0 > 0$. The asymptote (4.10) cuts the line (4.11) in the upwards direction (Fig.2). Thus the domain of stability for the elastic rod belongs to but is less than the domain of stability for the rigid rod. With $\mu = d = 0$, i.e., when no account is taken of the electric drive dynamics and the force F is the control, the curve (4.8), (4.10), (4.11) are identical, and so are the domains of stability.

Now take the most general case under condition (4.1) when $\gamma_2 \neq 0, T \neq 0$.

It can be seen from Eqs.(4.2) that, in the space of variables $\gamma_0, \gamma_1, \gamma_2$, the boundary of the domain of stability belongs to the plane $\gamma_2 = 0$. With $\gamma_2 = 0$, Eq.(3.6), (3.7) has the root $\lambda = 0$, and with $\gamma_2 < 0$, it has a real root $\lambda > 0$. Let us take a value $\gamma_2 > 0$, and construct the domain of stability in the plane of variables γ_0, γ_1.

Using arguments similar to those in the case (4.7), we can show that the boundary of the domain of asymptotic stability is given by parametric equations which are the same as (4.8) except for the presence of the term γ_2/ε^4 on the right-hand side of the second equation $(0 < \varepsilon < \varepsilon_1(m))$. As $\varepsilon \to 0$, the curve Γ approaches the γ_1 axis asymptotically, while as $\varepsilon \to \varepsilon_1$, it approaches the line shifted upwards relative to the line (4.10) by the value of this term. The domain of asymptotic stability bounded by the curve Γ is hatched in Fig.3.

The stability domain of the equilibrium $z = 0$ of system (2.4) is bounded by a branch of the hyperbola

$$\gamma_0 [(Td + \mu + m + 1)(\gamma_1 + d) - T(\mu + m + 1)\gamma_0] = (Td + \mu + m + 1)^2\gamma_2$$
$$(\gamma_0 > 0, \ \gamma_1 > -d, \ \gamma_2 > 0)$$

Its asymptotes are the γ_1 axis and the line Π (4.11). This domain is larger than the domain bounded by the curve Γ.

5. ***Stability when deformation signals are present.*** We renounce condition (4.1) and assume that

$$\sigma_1 \neq 0, \ x_1 = 0, \ \sigma_2 = \sigma_3 = \ldots = \sigma_N = 0 \tag{5.1}$$

The equations of the boundary of the domain of stability under condition (5.1) are obtained by adding to the left-hand side of the second equation of (4.2) the term

$$^1/_2\sigma_1\varepsilon^4 R_3(\varepsilon, 0, m) = \sigma_1\varepsilon^4 [\operatorname{sh}\varepsilon \sin\varepsilon + m\varepsilon Q_+(\varepsilon)]$$

Instead of the first equation in system (4.8) we now obtain

$$\gamma_0 = dT\varepsilon^4 + \mu\varepsilon^4 + \varepsilon^2 [\varepsilon R_2(\varepsilon, m) - {}^1/_2\sigma_1 R_3(\varepsilon, 0, m)]/R_1(\varepsilon, m) \tag{5.2}$$

With $0 < \varepsilon < \varepsilon_1(m)$ we have $R_3(\varepsilon, 0, m) > 0$. Hence, as may be seen from Eq.(5.2), with $\sigma_1 < 0$ the stability domain in the plane of γ_0, γ_1 is greater than the domain obtained under conditions (4.1), (4.7) by means of Eqs.(4.8). As $\sigma_1 < -\infty$, in spite of the presence of the delay T, the domain of stability obtained under conditions (4.7), (5.1) tends to the domain D (4.6) which is the domain of stability for an absolutely rigid rod with $\gamma_2 = 0, T = 0$. Thus, by introducing deformation signals into the feedback, we can increase the domain of stability. It would seem that, if the derivatives of these signals are also introduce into the feedback, the domain of stability can be further increased.

The domains of asymptotic stability can be constructed numerically by using our equations with specific values of the system parameters μ, m, d, T.

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FORCED OSCILLATIONS AND THE RADIATION OF SOUND BY A CIRCULAR PLATE INTERACTING WITH A FLUID*

S.N. BESHENKOV

A method is proposed for calculating forced oscillations and the acoustic radiation of a circular disc during its axially symmetric oscillations in an infinitely rigid baffle on the boundary of separation between fluid media. The dependence of the components of the deflection and the acoustic pressure on the excitation frequency as well as their distribution over the surface of the plate are investigated.

The proposed method is simpler than the use of expansions over orthogonal systems of functions /1, 2/. It leads to a finite resolvent system which contains the values of the acoustic pressure at a series of fixed points on the surface of the disc as unknowns. Compared with the finite-difference method** (Golovanov V.A., Muzychenko V.V., Peker F.N. and Popov A.L., Scattering and sonic emission by elastic shells in a fluid, Preprint No.261, Inst. Problem Mekhaniki Akad. Nauk SSSR, Moscow, 70 pp., 1985.) the proposed method enables one to attain the required accuracy using a smaller number of mesh points and leads to resolvent systems with better computational properties (according to the conditionality index). We also remark upon a method for determining the displacement potential of the fluid using a function of the deflection of the non-axially symmetrically oscillating disc /3/ and the results of experimental investigations of the hydroelastic oscillations of a disc /4, 5/.

Consider the forced oscillations of a circular disc which is clamped in an infinitely rigid baffle on the boundary of separation between fluid half spaces. Omitting the time factor $\exp(-i\omega t)$, we shall write the equation for the flexure of the disc taking account of the reaction of the acoustic media in the form

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